The Quantum Mechanical Measuring Process as a Scattering Phenomenon Inducing a Collective Coherent Motion

Manfred Requardt
Institut für Theoretische Physik, Universität Göttingen

Z. Naturforsch. **39a**, 1147 – 1159 (1984); received May 21, 1984

In this paper we want to discuss the quantum mechanical measuring process within the realm of many body quantum theory. Our starting point is to consider this process as a *special* scattering phenomenon where within one of the partners, i.e. the many body measuring device, a collective coherent motion is induced by the interaction with the microobject. We start our investigation with the many body system having a large but *finite* number N of degrees of freedom which is the real situation. We then study in detail what will happen in the limit $N \to \infty$, however emphasizing that this transition is actually only performed in the mind of the observer. This implies that certain tail events together with their phase correlations have to be truncated. We show that the dichotomy "pure state" versus "mixture" as outgoing scattering states will vanish in this limit in so far as it has no observable consequences provided one is only interested in the state of the microobject. Furthermore, we discuss the role of the observer, the notion of "event", the relation between single preparation and ensemble picture, and the so-called "reduction of the wave function" in the light of our approach, i.e. explaining the phenomena accompanying the measuring process in terms of many body quantum theory.

1. Introduction

Despite the many successes of quantum theory the dispute about same of the more fundamental questions of this now "classical" theory has never come to an end. As debates have not faded away for now over 50 years since the probably most widespread Copenhagen interpretation was formulated one cannot assume the attitude that this is only the well-known phenomenon that old habits of thinking will not sooner die out than the generation of scientists which supported these old ideas.

Quite on the contrary, this should give rise to the strong suspicion that there is still something unresolved in the epistemological foundation of the theory under discussion. As to quantum theory, one of the corner stones, and, on the other hand, one of the topics of hottest dispute is the "process of measurement", whatever this actually is to mean. One should, however, mention that this is not really an isolated body of problems since, as quantum theory is in a certain sense a selfconsistent interpretational scheme, there exist strong links to the other parts of the theory. To mention only a few, the problem of locality and the socalled "elements of reality" brought to light in the famous E-P-R paradox, the possibility of hidden variables etc.

In this paper we will be almost entirely engaged with what happens during the process of measurement. In particular, we want to analyse as concretely as possible the various facets of this complex process, thus showing that there are no mystic effects necessary to explain the phenomenon, as long as one is willing to accept quantum theory itself as the proper underlying structure. In our view, the mysteries have their roots mainly in the habit of not analysing the process of measurement in its *physical* details but instead starting immediately from one already highly abstracted, idealized scheme or the other. These extrapolations to the extremes usually bring about the strange effects and phenomena which are, on the other hand, not really present in the quantum-world.

To mention some points of controversy, there is the question

i) whether the state of the micro-object after the measurement is a pure one or a mixture,
ii) whether the acausal "instantaneous reduction" of the wave packet is a real phenomenon or is only taking place in the mind of the observer,

Reprint requests to Dr. M. Requardt, Institut für Theoretische Physik, Universität Göttingen, Bunsenstraße 9, D-3400 Göttingen.

0340-4811 / 84 / 1200-1147 $ 01.30/0. – Please order a reprint rather than making your own copy.

iii) at which place irreversibility does enter during the process of measurement, if there is any in quantum theory,

iv) what the role of the human observer actually is and, more specifically, in what sense the brain does function in accord with quantum theory or whether there are foreign elements in the mind are in conflict with it.

Before we will go into the details of these items and, in particular, before indicating what our own contribution to these questions is to be, we should perhaps mention the various different attitudes toward these problems, at least as far as we are aware of them.

The "classical" socalled Copenhagen interpretation is presented e.g. in Bohrs contribution in [1], resp. [2] or in the hand book article of Pauli [3]. Without going into any details at the moment, the state of the object is assumed to make a transition into a mixture after the measurement according to this interpretation; it is furthermore emphasized that there is no clear distinction between measuring apparatus and observer. Both are considered as "classical" systems the functioning of which (as measuring apparata) is classified *as ultimately not being derivable from quantum laws*. That is, quantum physics does need, in a certain sense, classical physics.

The axiomatic foundation of the measuring process was given by v. Neumann [4]. While the representation does mostly conform with the orthodox point of view there is, at least in our view, a certain deviation insofar as the state after the measurement is still assumed to be pure provided the ingoing state was pure. The reduction to a mixture is assumed to be accomplished by the brain of the observer, respectively "somewhere" between measuring apparatus and observer. This point has been further developed by Wigner (see e.g. the contribution in [5]) and represents a second attitude to these questions.

Quite the opposite standpoint has been occupied by G. Ludwig and his school. While the functioning of the measuring apparatus is analysed in concrete physical terms it is the intention of this approach to attribute the socalled irreversibility of the measuring process to some kind of *thermodynamical behaviour* of every "macroscopic apparatus" and is related to phenomena like ergodicity, approach to equilibrium etc., that is, the transition to a mixture as result of the measuring interaction is considered to be real. As references we want to cite [6–8]. In this context belongs also the analysis of Danieri et al. [9]. It is the intention of both approaches to downgrade the role of the observer.

While perhaps being a little bit stronger related to the classical point of view, the argumentation of Jauch (comp. e.g. [10]) is also along these lines with particular emphasis being laid on the point that the occurrence of an "event" is the true source of irreversibility which conforms with the classical point of view.

Still another type of explanation has been presented by D. Bohm, both concerning the mechanism of destruction of phase correlations and what the wave function does actually describe in case of a single preparation of a "microstate". An older but still illuminating source is [11], a more recent account is [12].

Furthermore, there are numerous contributions not yet mentioned which deal with one or the other facet of the problem, as e.g. [13–15], some of which can be found in [5], in which context the nearly exhaustive list of references in [16] should also be mentioned. Last but not least one should note that a concrete analysis of the measuring process has to be supplemented by a metastructure of a, so to say, syntactic and semantic character as has been given by e.g. Mittelstaedt resp. Ludwig and their schools (cf. e.g. [17]).

As to our personal motivation to discuss this body of questions anew; it is our aim to show that the quantum phenomena are actually independent of the observer and that the measuring process can be consistently explained within quantum theory itself.

We will start our investigation in Chapt. 2 with a critical analysis of the notion of *event* which serves exactly as the element of irreversibility in the usual orthodox quantum theory. This notion occurs on the level where also the individual microstate lives. In this context it has also to be clarified in what sense one can subsume these microstates under a certain wave function and to what extent these individuals reflect properties of the wave function they belong to.

While in Chapt. 2 the measuring apparatus will, in a preliminary, setting, be treated as the idealized system as it is introduced into the usual theory we shall discuss the interaction of microobject and measuring apparatus in Chapt. 3 in more realistic terms, namely as the interaction with a *many body* *system* resulting ultimately in the triggering of a coherent, collective process within a part of the measuring instrument. In this context we have also to take into account the finiteness of the *real* measuring devices. We want to show that this interaction can be consistently treated within the realm of the usual scattering theory where the particle number of one of the scattering partners approaches infinity. The mathematical aspects of this limiting process are discussed and are related to the underlying physics.

In section 4 we will discuss a couple of concrete measuring devices, in particular under the aspect where exactly the measurement does take place. Furthermore we concretize our claim, made precise in Chapt. 3, that at the core of the measurement process lies the triggering of a collective coherent many body process.

In the last section we will draw our conclusions from the preceding discussion. We will show that the whole measuring process can, in principle, be treated completely within the realm of quantum theory itself. Furthermore we show that in the limit: number of degrees of freedom of the measuring apparatus to infinity, the various descriptions resp. points of view coalesce, (e.g. "state" versus "mixture") as far as the observable content of the theory is concerned.

They turn out to be the alternative descriptions of one and the same underlying physical process. In particular, we want to show that the real process is independent of the human observer which has only the freedom to make a choice between two modes of description of one and the same body of underlying quantum phenomena.

Further items we hope to clarify are the socalled "reduction of the wave function", the axiom that measurements are mathematically represented by projections and whether a certain thermodynamic behavior of the measuring apparatus is really essential.

The central part of our paper consists of the Chapts. 3 and 5 while the Sects. 2 and 4 are of a complementary character.

2. The Preparation of the Microstate, the Notion of Event and their Relation to the Ensemble Picture

As to the definition of an "event" we want to cite Jauch ([10], p. 41 ff.), while this concept has been used, however, without much hesitation ever since the foundation of quantum mechanics:

"All of these experiments have one feature in common. They all end up with something that I shall call an event. We shall designate by event an *objectively* given phenomenon which has occurred in a physical system irrespective whether such a phenomenon has been observed by a conscious observer"....

"We shall distinguish an event from a *datum*. The latter is an event *magnified* to the level of human perception."

"What is significant, however, is that the Schrödinger equation for the evolution of states does not describe events. This can be seen from the fact that events do not occur with certainty."

As far as we can see from this and the examples mentioned in the paper, events are allowed to be of microscopic size. E.g. the first atom in a Geiger counter being ionized is the primary *event*, the following cascade is then the process of amplification. The scattering of a photon by the electron in the well-known electron microscope thought experiment is the event, etc. Here the amplification takes place in the retina and the visual part of the brain.

We want to confess that we feel a little bit uneasy with this concept since it carries with it a certain protophysical flavor. In this description events do objectively exist while the wave function describes the probability of occurrence of the various events under discussion. Whereas the detailed analysis of the functioning of the measuring apparatus shall be postponed to the following chapters the question suggests itself in what sense e.g. the atom being ionized in a Geiger counter differs from an arbitrary atom being ionized, i.e. this definition of "event" does not really depend on the measuring process proper. The measuring instruments are merely the devices to magnify and record the events, so that the observer can attribute a new wave function to the microstate.

We would like to give a brief analysis of this concept within a refined positivistic scheme, which we think is, on the other hand, also a realistic description. Quantum theory has taught us that in an individual preparation of a socalled "microstate" many "properties" of the microobject are "fixed" in a very weak and indirect sense. It may happen that e.g. the energy is just an eigenvalue, i.e. it remains well defined in the course of time. The classically relevant observables as e.g. position and momentum, having a continuous spectrum, on the other hand do neither remain well defined in the course of time nor are, due to the uncertainty relations, simultaneously given in the moment of preparation.

In the moment of preparation, t_0, all we can do is to attribute a wave function $\psi(t_0)$ to the microstate $s(t_0)$ expressed by $s(t_0)\ \varepsilon\ \psi(t_0)$, which combines all our "macroscopic" knowledge about the state. We have, however, no real access to the microstate $s(t_0)$ which may behave rather erratic in the course of time. The relation $s(t_0)\ \varepsilon\ \psi(t_0)$ however acquires its full meaning only after many preparations under constant macroscopic conditions at time t_0. The individual microstates subsumed under one and the same wave function $\psi(t_0)$ may, however, be microscopically different in each preparation.

Orthodox quantum mechanics seems to deny the very existence of an entity like $s(t)$. While our following analysis will not depend on this preassumption we would like to give a rough idea what we actually have in mind (this point was discussed and developed in greater detail in an earlier paper of the author [18]).

In our view the quantum fluctuations are a real, objectively existing, not further reducible phenomenon. That is, it is not our aim to introduce a hidden variables theory in the usual sense, i.e. reduction to a certain type of classical subdynamics. We rather think of quantum theory as a residual effect of a highly nonlinear *stochastic* interaction of the microobject with a strongly fluctuating vacuum structure. In contrast, however, to e.g. Brownian motion we consider the reaction of the object onto the state of the vacuum to be essential, so that the motion of the object will be *nonmarkovian*. An example from the realm of classical physics (however without fluctuations) would be a dislocation in a medium interacting with its own field of strains.

In this language of microstates the almost "classical" behavior of the measuring apparatus as a many body system is relatively easy to understand. While the states of its microscopic components $s_1, \ldots, s_N$, N large, are in the same manner poorly defined as in the case of the single microobject the "law of large numbers" has the effect that certain gross variables, which we call then "classical" are relatively stable under the time evolution. That is, while the fluctuations of the states $s_1, \ldots, s_N$, may be nonnegligible on the quantum level the relative fluctuations of certain *collective variables* can be considered as small on the macroscopic level.

Furthermore, we see from these considerations that it is an overidealization to attribute a definite wave function Ψ^M to the measuring apparatus. If the many body system is used as a measuring device only very few macroscopic observables are really under control, as e.g. positions of pointers, positions of the various center of masses of macroscopic subsystems, that is, we should better relate a whole class of many body wave functions $\{\psi^M\}$ to each controlled macrostate. The ψ^M's belonging to a certain class with label $\{i\}$ are then the quantum states being compatible with a certain fixed set of macroscopic parameters $\{i\}$ which we are actually interested in.

We want to argue now that the socalled "events", understood in a naive a priori sense, can never be observed. To this end we have to remember in what sense the functioning of the measuring apparatus is usually described. Taking as a typical example a Geiger counter the measuring process is described in a classical resp. semiclassical language. E.g. an incoming particle ionizes an atom and subsequently a cascade of ions is generated. From this we conclude that a particle has entered the counter.

The concreteness of this language is however deceptive. Since we have no real access to the microscopic regime we have only a rough idea of what really happened and try to describe the phenomena by resorting to a (semi)classical model world. The only (at the moment) consistent description can be given in terms of wave functions.

The microstate of the incoming particle $s(t)$ is then related to a wave function $\psi(t)$, $t < t_1$. The interaction and eventual ionization of an atom will be described by the microstate of a combined system belonging to a wave function $\Psi(r, R, t)$, $t \approx t_1$, r, R the particle resp. atom coordinates, which disintegrates into a sum of scattering states

$$\sum_i \psi_i(r, t) \cdot \Psi_i(R, t)$$

for $t \gg t_1$. In the course of time this produces a macroscopic collective effect, i.e. a cascade of more or less collectively moving ions resp. electron. Only *this* is the *real* phenomenon.

This again could be, on the one side, entirely described in terms of many body wave functions. It becomes an "event" only when the observer decides to describe the quantum phenomenon in terms of classical physics, e.g.: a large number of ions is moving toward the cathode. The soundness of this heuristic picture could now be given a consistency check by injecting a small but still macroscopic specimen into the counter which would record, expressed again in classical notions, a certain pressure, current etc.

To put it in a nut shell, on the one side there exists the quantum world and a description in terms of wave functions, on the other side we can apply a texture of (semi)classical notions to interpret the results. Only in this secondary phenomenological description do the "events" exist which in this derived scheme cause the effects registered in the measuring apparata. From a positivistic and at the same time realistic point of view only interactions of *microobjects* with macroscopic many body systems are real. Quantum mechanics makes just predictions about the probability with which the various macroscopic outcomes do show up after the interaction of the measuring instruments and the microobject has taken place. The only objective information we can derive from a measurement is new information about the probability of the outcomes of various future measurements on the system.

We can then underly this set of outcomes, in terms of a phenomenological classical description, a texture of certain "events", i.e. we can define: the object was roughly at the site R when a Geiger counter placed at R responded; we can furthermore check whether this texture provides a consistent description by employing further macroscopic measuring devices, but, in any case it remains a heuristic picture. The only thing which seems to objectively exist is the interaction of the microobject with the macroscopic measuring devices.

3. The Measuring Process Treated as an Interaction with a Special Class of Many Body Systems

In this section we want to treat the interaction of the microobject with the (in the orthodox terminology) "classical" measuring apparatus within the realm of interactions with a general many body system as it frequently occurs in many body resp. solid state physics. Since, evidently, not every interaction with e.g. a certain macroscopic system is a measurement it is our first task to classify this particular class of interactions resp. macroscopic systems. (Various classifications have already been given in the references, mentioned in the introduction.)

i) As a first step we have to remember that a concrete measuring device consists of a large but nevertheless finite number of microobjects, say N.

ii) In order to do its job a many body system which is designed to act as a measuring apparatus has to "amplify" quantum effects to macroscopic scales. To this end the measuring device is kept, in the beginning, in a certain controlled "metastable" state (not necessarily to be understood in a thermodynamic sense). The interaction with the quantum object then triggers a *coherent collective motion* of a still macroscopic subsystem, as e.g. a cascade of ions, the activation of a sensible grain on a photographic plate, the firing of certain nerve cells etc.

As long as we do not perform the limit $N \to \infty$ in our mind the measuring apparatus has, in principle, to be treated as a large quantum system. The transition to a socalled "classical behavior" in the limit is only an approximation suggested by the phenomenon of an induced collective motion of a large number of particles within the instrument. That is, for $N < \infty$, there is no reason for not approaching the problem within the realm of scattering theory.

To simplify the discussion in the following we introduce a couple of notational conventions: The wave function of the microobject s is $\psi(t)$; the many body wave function of the measuring device S^{M} is $\Psi^{\mathrm{M}}(t)$. Coordinates of microobjects s_i are denoted by r_i, R combines the whole set of degrees of freedom of the many body system S^{M}. R_i represent certain (possibly collective) coordinates of subsystems S_i^{M} of S^{M}. The observables to be measured are $A, B, \ldots$, in formulas the corresponding measuring instruments are abreviated by $\{A\}$, $\{B\}, \ldots$. Sometimes we want to speak of whole classes of wave functions which will be denoted by $[\Psi]$ with Ψ a certain representative.

One of the sources of constant confusion stems from the fact that frequently, when analysing the measuring process, the argumentation is not performed within a fixed epistomological frame. There are in principle two possibilities.

i) Either one treats *all* objects as quantum systems (possibly including parts of the brain of the observer),

ii) or one makes a clear (ad hoc) cut, motivated by practical purposes, in that one is only interested in the description of the wave function of the microobject proper and truncates the processes and the wave functions which occur resp. belong to the other devices employed during the experiment.

As a starting point we prefer the choice i). Furthermore, without loss of generality, we consider the initial state of the microobject to be always a pure one, i.e. which can be described by a wave function. While the complete wave function of the measuring instrument is never known (perhaps even in principle) it was already shown by v. Neumann [4] that we do not lose anything by assuming that, nevertheless, the apparatus is in the beginning also always in a certain pure state, i.e. $\Psi_0^{\mathrm{M}}\,\varepsilon\,[\Psi_0^{\mathrm{M}}]$, which is compatible with the few macroscopic parameters we have actually under control at the beginning of the measurement.

In an idealized experiment the measuring apparatus is usually expected to do the following: Assuming, for the time being, that the observable to be measured has a discrete spectrum with $\{\psi_i\}$ the corresponding eigenstates of the microobject we would like the combined system (object plus instrument) to make the following transition

$$\psi_i \otimes \Psi_0^{\mathrm{M}} \rightarrow \psi_i \otimes \Psi_i^{\mathrm{M}}\,, \tag{1}$$

with the l.h.s. roughly at $t_0 - \Delta t$, the r.h.s. at $t_0 + \Delta t$, provided we "observed" the corresponding macroscopic effect, labeled by the discrete parameter i, at the measuring apparatus in the interval $2\Delta t$ around t_0 (we consider only measuring devices which do not destroy the state ψ_i after the measurement, comp. e.g. [5]. Furthermore, as to the case of degeneracy of eigenvalues comp. [19]).

Remark: For reasons to be motivated later we prefer to restrict ourselves to macroscopic effects which consist (at least in the beginning) of a quantum mechanical collective *motion* of a certain subsystem of the many body system, e.g. described by the more or less coherent motion of a set of coordinates $\{r_1^i, \ldots, r_{N_i}^i\} =: R_i$ during a certain time interval Δt.

Within the scheme of quantum theory the state of the apparatus at time $t \sim t_0 + \Delta t$ would be described by a wave function ($R' := R \backslash R_i$):

$\Psi^{\mathrm{M}}(R', R_i;\; t_0 + \Delta t)$ with the peculiar property:

$$(\Psi^{\mathrm{M}} | P_i | \Psi^{\mathrm{M}}) \approx N_i \cdot \langle p \rangle_i\,, \tag{2}$$

where $\langle p \rangle_i$ is a characteristic microscopic mean momentum (a c-number) related uniquely to the "macroscopic" result $\{i\}$, P_i the collective momentum observable of a macroscopic portion of the system which is in a quantum state of collective motion

$$\left(P_i := \sum_i p_i,\; p_i = \frac{1}{m_i}(-i\,\nabla_i)\right).$$

While (1) seems to be a not further reducible starting point of the discussion there are already at this stage a couple of somewhat hidden problems. In order to start within an unproblematical Hilbert space we should better assume the apparatus to consist in the beginning of only finitely many particles N which implies also that the system occupies only a finite amount of space in $\mathbb{R}^3$.

Then, supressing for a moment the role of the human observer, we should expect a scattering phenomenon with an

i) ingoing state: $\psi_i \otimes \Psi_0^{\mathrm{M}}$,

ii) outgoing state: $\alpha\,\psi_i \otimes \Psi_0^{\mathrm{M}} + \beta\,\psi_i \otimes \Psi_i^{\mathrm{M}} + \sum_{i,k} \varepsilon_{jk}\,\psi_j\,\Psi_k^{\mathrm{M}}\,,$ (3)

where the dominant contributions are assumed to come from macroscopic situations where α) nothing happened, β) the expected effect occurred, γ) plus a certain small tail, expressed somewhat sloppily by assuming the $|\varepsilon_{jk}|$ to be small, in which all the scattering events are subsumed where the microobject s is scattered into a new state ψ_j, $j \neq i$, resp. emerges in the same state but with the "wrong" partner, Ψ_k^{M}, $k \neq i$. Furthermore, we want to remind the reader that the various vector states Ψ_i^{M} represent in principle whole classes of vectors which are grouped together according to the few characterizing macroscopic parameters we are interested in (i.e. $\{i\} \triangleq N_i \cdot \langle p \rangle_i$).

The input that the device should approximate a measuring apparatus is realized by assuming $|\beta|^2 \gg \sum_{j,k} |\varepsilon_{jk}|^2 =: \varepsilon_N^2$ where these tail events, occurring roughly with probability ε_N^2 arise from the *finiteness* of the many body system. In a general many body system, *not* designed to measure observables, *all* terms would usually contribute roughly with the same strength.

Remark: In this context we would like to mention an early observation by Wigner [20] who already emphasized that the naive assumptions about the functioning of measuring apparata are in conflict with quantum physics in cases where the observables to be measured are related to conservation laws. In our view (3) expresses the general situation as long as the apparatus is a system of finitely many degrees of freedom. It will, furthermore, be shown in the sequel that certain manipulations are perhaps a little bit problematical in the paper mentioned above, e.g. forming sums of Hilbert vectors which lie, strictly speaking, in different "superselection sectors".

With a general ingoing state $\psi = \sum c_i \psi_i$ we have a transition into:

$$\alpha \sum_i c_i \psi_i \Psi_0^{\mathrm{M}} + \beta \sum_i c_i \psi_i \Psi_i^{\mathrm{M}} + \sum \varepsilon'_{jk} c_j \psi_j \Psi_k^{\mathrm{M}} \tag{4}$$

with $|\beta|^2 \gg \sum |\varepsilon'_{jk}|^2 =: \varepsilon_N'^2$ being assumed.

Remark: We again emphasize that, in our view, (4) reflects the usual situation as long as the many body system being involved has only finitely many degrees of freedom (conservation laws only bring to light that (4) is the realistic ansatz).

The interesting point is now the passage to the limit: degrees of freedom, N, to infinity. As long as N is finite we learn from (4) that we can not be entirely sure that a macroscopic state Ψ_i^{M} is *strictly* related with a unique microstate ψ_i, there are small but non zero tail events with combinations of Ψ_i^{M}, ψ_k, $k \neq i$. There seems to exist even the possibility that nothing happened macroscopically, i.e. $\Psi_0^{\mathrm{M}} \to \Psi_0^{\mathrm{M}}$ while the microstate has changed, $\psi_i \to \psi_k$, $k \neq i$. More precisely, a certain representative of the class $[\Psi_0^{\mathrm{M}}]$ has been scattered into another element of the same macroscopic class. This transition will usually not be recorded by the observer (i.e. no collective motion). As one can learn from e.g. [20] and certain model calculations made, however, in another context (comp. e.g. [13, 14]), in the limit $N \to \infty$ the tail contributions become negligible, i.e. $\varepsilon'_N \to 0$, (a behavior $\lesssim N^{-1/2}$ has to be expected by simple probability theory), where β is assumed to be normalized to be of order one.

However, before we perform this limit it is useful to specify in more concrete terms the various many body wave function Ψ_i^{M} under discussion. We could have chosen, by brute force, an orthogonal system of states describing the finite system S^{M}. This would be the appropriate strategy if our systems were not designed to work as measuring devices. Each of our states Ψ_i^{M} is, however, assumed to be characterized by a specific collective motion of a certain subsystem S_i^{M}. That is, $\Psi_i^{\mathrm{M}}(R', R_i; t)$ has the coordinates subsumed under R_i well localized (with high probability) around some time dependent mean value

$$\langle R \rangle_i(t) := (\Psi_i^{\mathrm{M}}(t) \,|\, R_i \,|\, \Psi_i^{\mathrm{M}}(t)) \,. \tag{5}$$

As long as N_i is finite there will be, in general, a small but nevertheless finite overlap with the wave functions Ψ_k^{M}, $k \neq i$, with respect to the coordinates R_i, R_k being in collective motion of type $\{i\}$ resp. $\{k\}$. (Frequently the variables R_i, R_k will be identical, e.g. pointer coordinates but need not be exactly the same, e.g. the coordinates of the gas in a Geiger counter being involved in the motion.)

The various types of motion are assumed to be distinguishable by the localization in time which is different (for e.g. $\langle p \rangle_i \neq \langle p \rangle_k$, $i \neq k$). I.e., for N finite it is not a physically sound assumption to assume, from the outset, that $(\Psi_i^{\mathrm{M}} \,|\, \Psi_k^{\mathrm{M}}) = \delta_{ik}$, instead we should expect:

$$|(\Psi_i^{\mathrm{M}} \,|\, \Psi_k^{\mathrm{M}})| = \delta_{ik} \pm \hat{\varepsilon}_N \tag{6}$$

with an $\hat{\varepsilon}_N$ roughly of the order of the ε'_N in (4). In other words, for finitely many degrees of freedom there should still exist a small but non zero phase correlation.

For $N \to \infty$ two remarkable thing will happen.

i) $\hat{\varepsilon}_N$ will go to zero since in the scalar product (6) there are at least N_i small contributions coming from the coordinates $r_1, \ldots, r_{N_i}$ with small overlap of the various wave functions with resp. to these coordinates, which, on the other hand, has its origin in the different types of coherent motion $\{i\}$, $\{k\}$. These will add up roughly to a N_i-fold product of already small quantities. The same will happen with ε'_N in (4).

ii) The "limit states", $\{\Psi_i^{\infty}\}$, which will obviously leave any finite particle Hilbert space, will furthermore define *disjoint* representations of the algebra of observables of the many body system, in other words, they will define macroscopically different physical sectors s.t. Ψ_i^{∞}, Ψ_k^{∞}, $k \neq i$, will differ by the value of a superselection observable.

What do these observations imply for the measurements? Item ii) means the following. Assuming, for notational simplicity, that the coordinates involved in the various coherent processes are the same, i.e. $R_i = \{r_1, \ldots, r_{N_0}\} = R_k$ for all $\{i\}$, $\{k\}$, we can take as superselection observable in most of the cases the collective momentum $P = \sum_{i=1}^{N_0} p_i$ resp. $(1/N_0)\,P$. Then we shall have in the various macrostates:

$$(\Psi_i|P|\Psi_i) \approx N_0 \langle p \rangle_i, \quad (\Psi_k|P|\Psi_k) \approx N_0 \langle p \rangle_k. \quad (7)$$

With the characterizing mean momentum per particle

$$\langle p \rangle_i \neq \langle p \rangle_k \quad \text{for} \quad i \neq k.$$

Hence, with N which also implies $N_0 \to \infty$,

$$|(\Psi_i|P|\Psi_i) - (\Psi_k|P|\Psi_k)| \approx N_0 |\langle p \rangle_i - \langle p \rangle_k| \to \infty. \quad (8)$$

This is a situation well-known in the realm of many body physics (in particular in the regime of phase transitions). The various pure phases, $\{i\}$, can be classified by the classical quantities $\{\langle p \rangle_i\}$. It is then straightforward to prove that $\lim_{N_0 \to \infty} (1/N_0)\,P$ will become an element which commutes with all local observables which generate the algebra of quantum mechanical observables, $\mathscr{A}$, belonging to the many body system.

Hence $\lim_{N_0 \to \infty} 1/N_0\,P$ lies in the center of $\mathscr{A}$, its eigenvalues in the various irreducible representations of $\mathscr{A}$ with Hilbert spaces $\mathscr{H}_i$ are $\langle p \rangle_i$ where the $\mathscr{H}_i$'s are generated by applying $\mathscr{A}$ to an arbitrary representative of the class $[\Psi_i^{\mathrm{M}}]$. By this procedure we generate all states from a certain Ψ_i^{M} which are macroscopically indistinguishable from each other, i.e. $\mathscr{H}_i \equiv [\Psi_i^{\mathrm{M}}]$ as sets. To sum up our observations:

$$\lim_{N_0 \to \infty} [1/N_0\,P, A] = 0 \quad \text{for all} \quad A \in \mathscr{A} \quad (9)$$

(assumed to hold in an appropriate sense),

$$\lim_{N_0} 1/N_0 P \restriction \mathscr{H}_i = \langle p \rangle_i, \quad \mathscr{H}_i = \overline{\mathscr{A}\Psi_i^{\mathrm{M}}} = [\Psi_i^{\mathrm{M}}]$$

(As references where corrsponding phenomena are studied, however in another context, namely general many body physics, we want to mention e.g. the discussion of the BCS-theory by Haag in [18] resp. the book by Emch [19].)

We can now form a large Hilbert space

$$\mathscr{H} := \sum_i \mathscr{H}_i, \quad \mathscr{H}_i \text{ orthogonal to } \mathscr{H}_k \text{ for } i \neq k \quad (10)$$

but the representation of the quantum mechanical observables $\pi(\mathscr{A})$ also splits into a direct sum,

$$\pi(\mathscr{A}) = \sum_i \pi_i(\mathscr{A}), \quad [\pi_i(\mathscr{A}), \pi_k(\mathscr{A})] = 0 \quad \text{for} \quad i \neq k, \quad (11)$$

so that there remains no physical coupling between the various $\mathscr{H}_i$'s, at least within the realm of quantum theory.

What can be learned from this sequence of observations is that one can either work with a finite but large N, which is the real situation. Then one cannot escape the phenomena discussed in (3), (4), in particular there remains no room where one could escape regular scattering theory. Or one performs the limit $N \to \infty$, which is then rather an extrapolation of the real world, but in that case it makes not much sense to add e.g. vectors like Ψ_i^{M}, Ψ_k^{M}, $i \neq k$, which do not represent physically realizable situations. It is in this sense that we consider some of the manipulations in [20] to be a little bit problematical. We will continue our strategy to derive all the facets of the measuring process from quantum theory proper in Chapter 5.

4. The Analysis of Some Concrete Measuring Apparata

Before drawing our conclusions from the general analysis performed in Chapt. 3 it seems useful to discuss the functioning of some measuring instruments in more concrete terms. We shall start with, as we think, the most simple device, the Geiger counter.

i) (Geiger counter): It consists roughly of a neutral gas of atoms which is filled into the space between a cathode and an anode between which a certain potential difference exists. If a particle having enough energy enters the counter it may ionize one of the atoms. However, at least in principle, it may as well leave the counter without making an effect. Only in the case $N \to \infty$ (discussed in the preceding section) the probability to ionize an atom will approach one.

Assuming that an ionization has taken place of a certain but unknown atom at time t_0 being localized roughly at the point X in $\mathbb{R}^3$, the process has to be described by:

$$\psi(t) \times \Psi(t) \to \sum_{ij} \psi_i(t')\,\Psi_j(t'), \quad (12)$$

ψ, Ψ the wave functions of the object to be measured resp. of one of the atoms of the gas, $t < t_0$, $t' > t_0 + \Delta t$, $\{i, j\}$ labeling the various outgoing states after the interaction. (In a first approximation we have treated the wave function of the gas atom under discussion as independent of the wave functions of the other atoms.)

This description is in complete accord with quantum theory. Assuming now that the ingoing particle makes, in the average, only one ionization it will then leave the counter while the ionized atom will induce secondary processes which ultimately result in a cascade of ions, resp. electrons moving toward the cathode resp. anode. That is for $t'' \gg t_0$ we have the transition (keeping the point, X, of ionization for the moment fixed):

$$\sum_{ij} \psi_i(t')\, \Psi_j(t') \to \sum \psi_i(r-X; t'')\, \Psi_i^{\mathrm{M}}(R; X, t''). \tag{13}$$

The wave functions $\psi_i(r - X_i t'')$ will contain a factor of the shape $1/|r-X|$ for large times, Ψ_i^{M} are certain macroscopic wave function describing the coherent collective motion of the ions and electrons of the gas for large t'' and, as long as we do not average over the points X where the initial ionization happened, they will still carry a certain X-dependence. In principle the ionization could have taken place at any point X, that is, a definite point in space belongs to every single preparation of an ingoing state belonging to ψ. If we are however interested in the behavior of the full ensemble prepared in the state ψ we have furthermore to integrate over the coordinate X. This point will be further discussed in Section 5.

This displays once more the dualistic feature of a many body system used as a measuring instrument. In principle one could localize the point of the first ionization in space but this would, as far as we can see, make the system unable to function as a measuring apparatus proper. These phenomena were already discussed at length, a little bit in another context, in the beautiful papers of Bohr (comp. e.g. [1]). Furthermore one can see from the above the dichotomy between a description either in terms of single preparations or ensembles.

We shall finish this section by discussing a couple of other typical experimental set ups which are chosen to clarify another facet of the measuring process. Sometimes it is claimed that a measurement can, in principle, be performed by employing another microobject without inducing a macroscopic collective process. As examples we choose the

ii) (measurement of position by a microscope, resp. the Stern-Gerlach experiment): As to the former well-known thought experiment, it is sometimes claimed that the position of e.g. an electron is measured by a single scattered photon (to avoid problems with the highly relativistic behavior of photons we simplify the discussion by replacing the photon by a non-relativistic particle having a nonzero mass (comp. e.g. [6]).

After the scattering event has taken place the state of the scattered particle is in the lowest approximation a certain spherical wave containing a factor proportional to $\exp(ik(r-X)/|r-X|$, X the "point" of scattering, which is then deflected by a macroscopic lense (in other words, a many body system) making in the end a collective effect on the retina. Only the combination lense plus retina/brain makes a position measurement possible, that is, the "reduction" of the spherical wave is brought about again by a many body system and not by the deflection of a single photon.

The Stern-Gerlach experiment is also sometimes mentioned as a case in point concerning the measurement of a quantity by employing only a *single* additional particle (the atom itself which carries the spin is claimed to serve as a classical pointer). But we think this point of view is not correct. The first point to mention is that the exterior classical magnetic field, which plays actually the role of the lense in the foregoing example, is a many body system in disguise (consisting of many photons generated e.g. by an electric current).

If we do not undertake special measures the wave function will not be reduced when leaving the region of interaction. Instead it will be of the form:

$$\Psi = f_+(z, t)\, v_+ + f_-(z, t)\, v_- \tag{14}$$

with $f_\pm$ describing the vertical localization in space, $v_\pm$ the spin orientation. Furthermore, as long as the atom has a finite mass, everything will apply what we have said in the preceding section, i.e. f_+, f_- will have a finite overlap for all times (for the details of formula (14) see e.g. [6] resp. [11] whereas the interpretation differ from each other).

That (14) is the correct description can be shown by recombining the two terms in (14) into a wave function which does not develop two strongly separated packets in the course of time. On the other hand one can, as was lucidly discussed by N. Bohr in another context in [1], prepare the magnets in such a manner that the spin measurement is performed during the passage through the magnetic field. In this case the derivation leading to formula (14) will not longer hold. The position of the magnets are then no longer fixed in space in order that a collective process can be initiated within the magnetic device.

In the usual case (i.e. which leads to (14)), the spin measurement is carried out not earlier as a sensible grain has been activated in the detecting screen. In other words we do not follow the reasoning that the atom itself is already the classical coordinate of the apparatus, the crucial step is again the collective process in one of the silver granules.

5. "State" versus "Mixture", the Acquiring of Information and the Role of the Observer

In this final chapter we shall draw our conclusions from the observations presented in the preceding sections. The remaining items to discuss are i) whether the outgoing state is a pure vector state or a mixture and to what extent this is a question of observable relevance or, rather, a free decision made by the mind of the observer. ii) How the observer acquires what we usually call "information" by employing the measuring apparata. iii) Whether, resp., in what sense a "reduction" of the wave packet is accomplished by a measurement.

As long as the number of degrees of freedom of the measuring apparatus is finite we have seen in Sect. 3 that we have no good reason to not consider the combined system apparatus plus microobject to be in a vector state after the interaction has taken place. We should be rather in a situation as described by formulas (4) to (6). Hence, for finitely many degrees of freedom quantum theory leaves no room for transitions into mixtures.

This led Ludwig et al. to introduce some sort of thermodynamic behavior of the measuring apparata as the source of destruction of phase correlations and irreversibility. But we showed in Sect. 3 that the primary macroscopic collective process initiated by the interaction with the many body system is not, first of all, a transition into a thermodynamical equilibrium state but, rather, a time dependent state of a *coherent motion* of a large number of microobjects.

The phenomenon of dissipative motion, i.e., the ultimately coming to rest of e.g. a pointer in a certain position is, in our view, only a nice side property of thermodynamic systems when being employed as measuring devices which makes the quantification of observations easier but we do not think that it is the essential ingredient. We maintain that the measuring process is, basically, a temperature zero phenomenon and that the role of temperature shows up only in a secondary step, namely to guarantee that the collective motion will ultimately die out.

Following the line of reasoning developed in Sect. 3 the first step of abstraction beyond the real quantum world consists of making the transition $N \to \infty$. This is a purely *mental* act and implies that the tail events mentioned in Chapt. 3, which occur in the real world, be truncated. That is, in idealized form the measurement of an observable A can be described as follows:

Let ψ_0 be a superposition of eigenstates of the observable A, i.e. $\psi_0 = \sum c_i \psi_i$ we have:

$$\psi_0 \times \Psi_0^{\mathrm{M}} \xrightarrow{\{A\}} \alpha\, \psi_0\, \Psi_0^{\mathrm{M}} + (1-\alpha) \sum_{i=1}^{\infty} c_i\, \psi_i\, \Psi_i^{\mathrm{M}}, \tag{15}$$

where $\alpha \neq 0$ incorporates the possibility that nothing happened in the scattering process. (In the real world the measuring apparata occupy only a finite amount of space. It depends on the spatial extension of the ingoing wave function and on the type of interaction whether we can approximately α assume to be almost zero or not.)

We want to emphasize that (12) is a mathematical well defined limit description of the real quantum world arrived at by performing the transition $N \to \infty$. In particular, it has the merits to be observer independent, i.e. we do not need epistomological elements being foreign to quantum theory proper. On the other hand the human observer can make a mental decision to shift to another mode of description which is for $N < \infty$ only an approximation of the real situation described in Sect. 3 but will for $N \to \infty$ yield the same *observational* results about the microobject as the complete microscopic description which, however, describes the full experimental set up including the measuring devices. How this comes about will be explained in the following.

In this second mode of description the analogous transition as compared with (15) is described in an ad hoc manner as:

$$P_{\psi_0 \Psi_0^{\mathrm{M}}} \xrightarrow{\{A\}} \sum_i |c_i|^2 \, P_{\psi_i \Psi_i}, \tag{16}$$

where the P's are the projectors onto the states occurring in the subscripts, the r.h.s. representing now a mixture. In the orthodox theory formula (16) is considered to be almost self-evident. But this can only be accomplished by introducing the observer as an ad hoc and foreign element into the description, i.e., he acts in a "classical" manner which is considered to be ultimately inconsistent with quantum theory. It is argued that he "observes" the outcomes of the measuring interaction and then groups together the various microstates into subensembles according to the type of macroscopic effect. This is then exactly the transition into a mixture. We will prefer to approach these problems by relying on the results of Section 3.

Formulas (4)–(6) describe the real situation i.e. an outgoing state having certain tail events and small but non zero phase correlation between the various terms. In the limit $N \to \infty$ the expressions (7)–(11) show that the various macroscopic limit states Ψ_i^{M} will lie in disjoint Hilbert space sectors, $\mathscr{H}_i$, of the large space $\mathscr{H}$. This implies that sums of states lying in disjoint sectors do no longer make a physical sense (as in the case of superselection observables they are not *physically* realizable) but they can be considered to be mutually orthogonal.

A further important point to mention is that, since this description is "observer free" the eventual process of regrouping the outgoing individual states according to the macroscopic effect they induced in the measuring system has to be done again by a large quantum system and not by an ad hoc introduced exterior classical observer. The measurement of an observable A with resp. to an ingoing wave function ψ_0 is then described by (assuming $\alpha = 0$ in (15)):

$$\langle A \rangle = \sum_i |c_i|^2 \, a_i \tag{17}$$

with $|c_i|^2$ the frequency of the macroscopic outcome $\{i\}$, α_i the eigenvalue related to this macroscopic phenomenon.

In orthodox quantum theory the reason why it makes sense to say that one has actually measured a property of the microsystem is exhibited by the fact that an immediate consistency measurement of A performed on the outgoing state will yield the same result. In our description of this measurement we have to place another measuring instrument $\{A'\}$ measuring the same observable A in the immediate spatial neighborhood of $\{A\}$. We already see from this that in principle the finiteness of the various apparata is unavoidable as well as the corresponding complications mentioned above. Suppressing these side effects we would get (for reasons of notational simplicity we will omit frequently the tensor product "$\times$" in the following):

$$\begin{aligned}&\left(\sum c_i \psi_i \Psi_i^{\mathrm{M}} \,|\, A \,|\, c_i \psi_i \Psi_i^{\mathrm{M}}\right) \\ &\quad = \sum_i |c_i|^2 (\psi_i \,|\, A \,|\, \psi_i) \cdot (\Psi_i^{\mathrm{M}} \,|\, \Psi_i^{\mathrm{M}}) = \sum |c_i|^2 \, a_i ,\end{aligned} \tag{18}$$

since $(\Psi_i^{\mathrm{M}} | \Psi_j^{\mathrm{M}}) = \delta_{ij}$ and a wave function leaving the second apparatus:

$$\sum c_i \psi_i \Psi_i^{\mathrm{M}} \Psi_i'^{\mathrm{M}}, \quad (\Psi_i^{\mathrm{M}} | \Psi_j'^{\mathrm{M}}) = 0 \quad \text{for all } i, j, \tag{19}$$

where we have to assume the many body wave functions Ψ_i^{M} to be essentially localized within $\{A\}$, $\Psi_i'^{\mathrm{M}}$ within $\{A'\}$ with zero spatial overlap of $\{A\}$, $\{A'\}$. Again we see that the result would be the same in the case of a mixture instead of a pure state.

If one wants to measure at first the observable A and afterwards the observable B (making again the necessary assumption that a measurement of B will not influence the many body wave functions of the apparatus $\{A\}$, i.e. that $\{B\}$ interacts only with the microobject) we get (with $\psi_{i,A}$, $\psi_{i,B}$ the systems of eigenfunctions of A, B):

$$\begin{aligned}&\left(\sum c_i \psi_{i,A} \Psi_{i,A}^{\mathrm{M}} \,|\, B \,|\, \sum c_i \psi_{i,A} \Psi_{i,A}^{\mathrm{M}}\right) \\ &\quad = \sum |c_i|^2 (\psi_{i,A} \,|\, B \,|\, \psi_{i,A}) (\Psi_{i,A}^{\mathrm{M}} \,|\, \Psi_{i,A}^{\mathrm{M}}) \\ &\quad = \sum |c_i|^2 (\psi_{i,A} \,|\, B \,|\, \psi_{i,A})\end{aligned} \tag{20}$$

and a state leaving $\{B\}$:

$$\sum_{i,j} c_i (\psi_{j,B} \,|\, \psi_{i,A}) \, \psi_{j,B} \, \Psi_{i,A}^{\mathrm{M}} \Psi_{j,B}^{\mathrm{M}} \tag{21}$$

with

$$(\Psi_{i,A}^{\mathrm{M}} \,|\, \Psi_{j,B}^{\mathrm{M}}) = 0 \quad \text{for all } i, j .$$

The observed mean value (20) is again the same as in the case of a mixture.

So far we see from this analysis that we can never test whether the state leaving the apparatus is a pure one or a mixture as long as we do not interfere in a manner which goes beyond the scope of quantum theory proper. The argumentation presented in Sect. 3 supplies us with sufficiently strong orthogonality conditions within the scheme of quantum theory s.t. all the observable results will be the same in both descriptions.

To make now clearer what we mean by the mentioned foreign elements of description we have to come now to one of the most delicate points in this context, namely how the observer manages to acquire information about the microobject. In the spirit of Sect. 3 we have to argue as follows: In principle the observation of the various outcomes of the measuring interaction with a certain apparatus, say $\{A\}$, i.e. the recording of the various types of coherent motion can be done by a suitable computer.

Therefore it is not primarily a mind-matter problem. the computer is then simply just another, in reality a finite, macroscopic quantum system which, however, does not interact, as in (20), (21), with the microstates ψ_i but with the many body states $\Psi^{\mathrm{M}}_{i,A}$. To be more specific, the instrument $\{B\}$ will test the mean momentum $\langle p\rangle_i$ characterizing the collective state $\Psi^{\mathrm{M}}_{i,A}$ and will then switch into a corresponding collective inner state $\Psi^{\mathrm{M}}_{i,B}$, i.e. the resulting outgoing state will be:

$$\sum c_i \psi_i \Psi_{i,A} \Psi_{i,B} \tag{22}$$

where the $\Psi_{i,A}$'s, $\Psi_{i,B}$'s are localized in disjoint regions of space.

In a next step one might prefer to have the various groups of microstates belonging to a ψ_i ultimately localized in spatially well separated regions of space, this would be the spatial analog as to what the mind of the observer does when it groups together the various microstates according to the observed measuring results. To this end one has to couple another system $\{C\}$ with $\{B\}$ which then scatters the various ψ_i's into the well separated regions $\mathcal{O}_i$ of space. But it is unavoidable that this has the effect that also $\{C\}$ makes a corresponding transition into a state $\Psi^{\mathrm{M}}_{i,C}$. So in the end we arrive at an overall state (t large):

$$\sum c_i \psi_i(t) \Psi_{i,A}(t) \Psi_{i,B}(t) \Psi_{i,C}(t), \tag{23}$$

where $\{C\}$ is in a state $\Psi_{i,C}$ when the microobject is localized in the region $\mathcal{O}_i$ with high probability for large t.

In other words, what we have accomplished entirely within the realm of quantum theory is a splitting of the state of the microobject into widely separated pieces located almost surely in $\mathcal{O}_i$. When we decide to do experiments in the region $\mathcal{O}_i$ we can be almost sure to have the quantum object in a state ψ_i, i.e. we have accomplished the same physical situation as in the case where a human observer makes the various decision.

Now let us assume that the apparatus $\{B\}$ in (22) resp. (23) is a part of the brain which performs the same steps as a computer. Observation of the state $\Psi_{i,A}$ of the apparatus $\{A\}$ means now that the inner state of $\{B\}$ switches into a corresponding $\Psi_{i,B}$ (as a result of a certain ineraction). This is an overall quantal many body state of the affected parts of the brain with the corresponding nerve cells displaying a certain coherent behavior as discussed in Chapter 3. This coherent state is then transformed into an ordered arrangement of impressions, thoughts etc., which are however on the quantum level again represented by nothing but collective quantum states of other parts of the brain. There is in fact no urgent need to leave this regime of many body quantum theory to understand what is going on.

If we remember now what we said in Sect. 3 the eventual transition to a description in terms of mixtures is nothing but a truncation of the various macrostates, e.g. $\Psi_{i,A}$, $\Psi_{i,B}$, $\Psi_{i,C}$ from the primary description. If the observer prefers to make a simplification of his theoretical description because he is mainly interested in the wave functions $\{\psi_i\}$ of the microobject he can make the free decision to forget about the real tensor product structure of the outgoing states. The observable content will remain the same, as long as he is only interested in the quantum object, when he describes the situation by means of mixtures built solely from the states $\{\psi_i\}$ with the macrostates $\{\Psi^{\mathrm{M}}_i\}$ being truncated.

The last topic we want to address in this section is the socalled "reduction of the wave packet" as it is called in the orthodox theory. We will rely on the preparatory remarks already done in Chapt. 4 and choose as a typical example the measurement of position by a Geiger counter. As can be seen from formula (13) the representation of the measuring process for each *single preparation* of an ingoing state ψ does not really reflect a situation where after the measurement the wave function is reduced to $\chi_{\mathcal{O}} \cdot \psi$, $\chi_{\mathcal{O}}$ the characteristic function of the region, $\mathcal{O}$, being occupied by the Geiger counter.

Rather, the outgoing scattered components contain a term $\sim 1/|r-X|$, X the point of ionization of a gas atom. This exhibits once more a certain dichotomy between the single preparation of states and the ensemble picture. That is, the observer has usually no access to these points X. If many micro-states have been prepared with wave function ψ, the sites $\{X_i\}$ where the first ionization happened are more or less at random. It est, if we switch to the ensemble picture we effectively average over the sites $\{X_i\}$ in formula (13).

With the ψ_{X_i} being roughly of the shape

$$\psi_{X_i} \sim 1/|r - X_i| \cdot \exp\{i\,k(r - X_i)\} \tag{24}$$

and for N, the number of gas atoms to infinity s.t. usually the relevant $|k|$'s are large as compared with the mutual distances of the gas atoms, $|X_i - X_j|$, we shall observe, in a description by ensembles and then mixtures, an outgoing distribution of positions of the form:

$$\sum_i |\psi_{X_i}|^2 (r - X_i)\,. \tag{25}$$

For a large ensemble the X_i-dependence will drop out and the outgoing wave used in a description by mixtures can then be approximated by $\chi_{\mathscr{V}}(r) \cdot \psi(r)$ since:

$$\sum_i |\psi_{X_i}|^2 (r - X_i) \to \chi_{\mathscr{V}}(r)\,|\Psi|^2(r)\,. \tag{26}$$

The discussion of other measuring setup would run along similar lines. Id est as for the other items already discussed, the reduction of the wave function appears to be also dependent on the mode of description the observer chooses. It seems to belong more to a description by mixtures while in the many body description its presence is more of an implicit nature.

[1] P. A. Schilp (ed.), Albert Einstein, Philosopher-Scientist, Libr. of Living Philosophers, Inc., Evanston, Illinois 1949.
[2] W. Heisenberg, Physik-Prinzipien der Quantentheorie, Band I, Mannheim 1958.
[3] W. Pauli, in: Handbuch der Physik, ed. S. Flügge, Springer, Berlin 1958.
[4] J. v. Neumann, Math. Grundlagen der Quantenmechanik, Springer, New York 1968.
[5] E. Wigner, in: Scoul. Int. di Fis. "Eurico Fermi", Il Corio Fondam. di mecc. quantistica. Academic Press, New York 1970.
[6] G. Ludwig, Die Grundlagen der Quantenmechanik, Springer, Berlin 1954.
[7] G. Ludwig, in [5].
[8] G. Ludwig, Einführung in die Grundlagen der Theoretischen Physik, Bd. 4, Vieweg, Braunschweig 1979.
[9] A. Danieri, A. Loinger, and G.M. Prosperi, Nucl. Phys. 133, 297 (1962).
[10] J. M. Jauch, in [5].
[11] D. Bohm, Quantum Theory, Constable and Comp., London 1954.
[12] D. Bohm and J. Bub, Rev. Mod. Phys. **38,** 453 (1966).
[13] K. Hepp, Helv. Phys. Act. **45,** 237 (1972).
[14] G. G. Emch and B. W. Whitten-Wolfe, Helv. Phys. Act. **49,** 45 (1976).
[15] H. Araki and M. M. Yanase, Phys. Rev. **120,** 622 (1960).
[16] G. Reece, Int. J. Theor. Phys. **7,** 81 (1973).
[17] P. Mittelstaedt, Festschrift, Grundlagenprobleme der modernen Physik (J. Nitsch, J. Pfarr, and E.-W. Stachow, eds.), 1979.
[18] M. Requardt, Preprint Göttingen.
[19] G. Lüders, Ann. Phys. **8,** 32 (1951).
[20] E. P. Wigner, Z. Physik **133,** 101 (1952).
[21] R. Haag, Nuovo Cim. **25,** 287 (1962).
[22] G. Emch, Algebraic Methods, in: Stat. Mechanics & Field Theory. John Wiley, New York 1972.